Identification de modules fonctionnels par l'analyse topologique d'un réseau de corégulation

> Etienne Delannoy Pierre Latouche

> > Netbio, 2019

Biological context

Multiple biotic and abiotic stresses impacting plant growth

Coordinated response to stresses in general ?

Numerous single stress transcriptomic data sets available \rightarrow Stress gene co-expression network

Coexpression analyses of 18 stress responses

h: 387 comparisons in 18 stress categories: 9 biotic and 9 abiotic

http://tools.ips2.u-psud.fr/CATdb

Zaag R *et al.,* Nucleic Acids Res, 2015

From stress coexpression clusters to stress coregulation gene network

Coexpression clusters for each category of stress

Integration

Occurrence of pairs of coexpressed genes conserved in several stresses among the 18 considered stress categories

Coexpression network

Coregulation network

1) Compared with random networks, only edges providing a FDR<1% were kept

Filters

2) Only genes involved in triangles were considered as co-regulated

Arabidopsis stress co-regulation network

4476 genes and 56487 co-regulation links

86% of the co-regulation links are supported by both biotic and abiotic stresses

- Scale-free network Density = 0.006
	- Transitivity= 0.54
	- \rightarrow Biological network
	- $→$ **Presence of gene clusters**

Stochastic Block Model (SBM) [\[WW87,](#page-0-0) [NS01\]](#page-0-1)

 \blacktriangleright *Z_i* independent hidden variables :

$$
\blacktriangleright Z_i \sim \mathcal{M}(1, \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_K))
$$

 \blacktriangleright *Z*_{*ik*} = 1 : vertex *i* belongs to class *k*

▶ *X*|*Z* edges drawn independently :

$$
X_{ij}|\{Z_{ik}Z_{jl}=1\}\sim\mathcal{B}(\pi_{kl})
$$

 \blacktriangleright A mixture model for graphs :

$$
X_{ij} \sim \sum_{k=1}^K \sum_{l=1}^K \alpha_k \alpha_l \mathcal{B}(\pi_{kl})
$$

Maximum likelihood estimation

I Log-likelihoods of the model :

- \triangleright Observed-data : $\log p(X|\alpha, \pi) = \log \{\sum_{Z} p(X, Z|\alpha, \pi)\}\$ \hookrightarrow *K^N* terms
- \triangleright Expectation Maximization (EM) algorithm requires the knowledge of $p(Z|X, \alpha, \pi)$

 $p(Z|X,\alpha,\pi)$ is not tractable (no conditional independence)

Variational EM Daudin et al. [\[DPR08\]](#page-0-2)

Maximum likelihood estimation

► Log-likelihoods of the model :

 \triangleright Observed-data : $\log p(X|\alpha, \pi) = \log \{\sum_{Z} p(X, Z|\alpha, \pi)\}\$ \hookrightarrow *K^N* terms

 \blacktriangleright Expectation Maximization (EM) algorithm requires the knowledge of $p(Z|X, \alpha, \pi)$

Problem

 $p(Z|X,\alpha,\pi)$ is not tractable (no conditional independence)

Variational EM Daudin et al. [\[DPR08\]](#page-0-2)

Maximum likelihood estimation

► Log-likelihoods of the model :

 \triangleright Observed-data : $\log p(X|\alpha, \pi) = \log \{\sum_{Z} p(X, Z|\alpha, \pi)\}\$ \hookrightarrow *K^N* terms

 \blacktriangleright Expectation Maximization (EM) algorithm requires the knowledge of $p(Z|X, \alpha, \pi)$

Problem

 $p(Z|X,\alpha,\pi)$ is not tractable (no conditional independence)

Variational EM Daudin et al. [\[DPR08\]](#page-0-2) Graphical model and moral graph

Moral graph of SBM

Model selection

Criteria

Since $\log p(X|\alpha, \pi)$ is not tractable, we *cannot* rely on :

$$
AIC = \log p(X|\hat{\alpha}, \hat{\pi}) - M
$$

►
$$
BIC = \log p(X|\hat{\alpha}, \hat{\pi}) - \frac{M}{2} \log \frac{N(N-1)}{2}
$$

ICL

Biernacki et al. [\[BCG00\]](#page-0-3) \hookrightarrow Daudin et al. [\[DPR08\]](#page-0-2)

Variational Bayes EM ,! *ILvb*

Latouche et al. [\[LBA12\]](#page-0-4)

Others McDaid et al. [\[MDMNH13\]](#page-0-5)

52 communities of 21 to 351 genes

Stability of the communities?

2674 genes in 43 communities describe the common response to stresses

Cross-validation procedure

- For each stress category, create a network from the 17 others
- Find communities using mixture of graphs
- Comparison of these 18 results with the network built from all the categories

Most communities with GO enrichments

Functional validation of the communities

Functional validation of the communities

Pictures by S. Domenichini

Functional validation of the communities

Functional validation of the communities

Phenotyping of T-DNA mutants of 8 genes with unknown function

Germination speed on 150mM NaCl

>3 biological replicates with 1 seed lot

Functional validation of the communities

 2.0 Heat stress *** *** ** 3 days-old etiolated * Hypocotyle length (cm) Hypocotyle length (cm) *** ** seedlings incubated at *** *** $\frac{10}{1}$ 44°C for 1h30 before 3 days of normal conditions. 3 independent exp. $\frac{0}{1}$ 0.5 Col0 control S \ddot{c} Ċ. $\mathbb C$ $\mathbb S$ $\mathbf C$ C S C. stressedC

> Phenotyping of T-DNA mutants of 8 genes with unknown function

Functional validation of the communities

The backbone of plant stress response

Identification of stress specific communities within the network

Introduction

FIGURE – An (hypothetic) email network between a few individuals.

Introduction

FIGURE – A typical clustering result for the (directed) binary network.

Introduction [\[BLZar\]](#page-0-0)

Figure – The (directed) network with textual edges.

Introduction [\[BLZar\]](#page-0-0)

FIGURE – Expected clustering result for the (directed) network with textual edges.

Context and notations

We are interesting in clustering the nodes of a (directed) network of *M* vertices into *Q* groups :

 \blacktriangleright the network is represented by its *M* \times *M* adjacency matrix *A* :

 $A_{ij} =$ $\left\{ \right.$ \vert 1 if there is an edge between i and j 0 otherwise

If $A_{ij} = 1$, the textual edge is characterized by a set of D_{ij} documents :

$$
W_{ij}=(W_{ij}^1,...,W_{ij}^d,...,W_{ij}^{D_{ij}}),
$$

 \blacktriangleright each document W_{ij}^d is made of N_{ij}^d words :

$$
W_{ij}^d = (W_{ij}^{d1}, ..., W_{ij}^{dn}, ..., W_{ij}^{dN_{ij}^d}).
$$

Modeling of the edges

Let us assume that edges are generated according to a SBM model :

• each node *i* is associated with an (unobserved) group among *Q* according to :

 $Y_i \sim \mathcal{M}(\rho)$,

where $\rho \in [0, 1]^Q$ is the vector of group proportions,

 \blacktriangleright the presence of an edge A_{ij} between *i* and *j* is drawn according to :

$$
A_{ij}|Y_{iq}Y_{jr}=1\sim\mathcal{B}(\pi_{qr}),
$$

where $\pi_{qr} \in [0, 1]$ is the connection probability between clusters *q* and *r*.

Modeling of the documents

The generative model for the documents is as follows :

lace each pair of clusters (q, r) is first associated to a vector of topic proportions $\theta_{ar} = (\theta_{ark})_k$ sampled from a Dirichlet distribution :

 $\theta_{ar} \sim \text{Dir}(\alpha)$,

such that $\sum_{k=1}^{K} \theta_{qrk} = 1$, $\forall (q, r)$.

 \blacktriangleright the *n*th word W_{ij}^{dn} of documents *d* in W_{ij} is then associated to a latent topic vector Z^{dn}_{ij} according to :

$$
Z_{ij}^{dn}|\left\{ A_{ij}Y_{iq}Y_{jr}=1,\theta\right\} \sim \mathcal{M}\left(1,\theta_{qr}\right) .
$$

 \blacktriangleright then, given Z^{dn}_{ij} , the word W^{dn}_{ij} is assumed to be drawn from a multinomial distribution :

$$
W_{ij}^{dn}|Z_{ij}^{dnk}=1 \sim \mathcal{M}(1,\beta_k=(\beta_{k1},\ldots,\beta_{kV}))
$$

where *V* is the vocabulary size.

STBM at a glance...

Figure – The stochastic topic block model.

STBM at a glance...

Figure – The stochastic topic block model.

Inference

The full joint distribution of the STBM model is given by :

 $p(A, W, Y, Z, \theta | \rho, \pi, \beta) = p(W, Z, \theta | A, Y, \beta) p(A, Y | \rho, \pi)$.

A key property of the STBM model :

- \blacktriangleright let us assume that *Y* is observed (groups are known),
- it is then possible to reorganize the documents $D = \sum_{i,j} D_{ij}$ documents *W* such that :

$$
W = (\tilde{W}_{qr})_{qr} \text{ where } \tilde{W}_{qr} = \left\{ W_{ij}^d, \forall (d, i, j), Y_{iq} Y_{jr} A_{ij} = 1 \right\},\
$$

- ightharpoontriangleright since all words in \tilde{W}_{qr} are associated with the same pair (*q*,*r*) of clusters, they share the same mixture distribution,
- **If** and, simply seeing \tilde{W}_{ar} as a document *d*, the sampling scheme then corresponds to the one of a LDA model with $D = Q^2$ documents.

Inference

Given the above property of the model, we propose for inference to maximize the complete data log-likelihood :

$$
\log p(A, W, Y | \rho, \pi, \beta) = \log \sum_{Z} \int_{\theta} p(A, W, Y, Z, \theta | \rho, \pi, \beta) d\theta,
$$

with respect to (ρ, π, β) and $Y = (Y_1, \ldots, Y_M)$.

7 topics

bacteria

Nitrogen

Conclusions of a biologist

Conclusions of a biologist

Acknowledgements

IPS2

Marie-Laure Martin-Magniette Guillem Rigaill Rim Zaag Nathalie Rézé The IPS2 transcriptomic platform The GNet team