Identification de modules fonctionnels par l'analyse topologique d'un réseau de corégulation

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### **Biological context**

# Multiple biotic and abiotic stresses impacting plant growth



Coordinated response to stresses in general ?



Numerous single stress transcriptomic data sets available → Stress gene co-expression network

### Coexpression analyses of 18 stress responses

1: 387 comparisons in 18 stress categories: 9 biotic and 9 abiotic

http://tools.ips2.u-psud.fr/CATdb



Zaag R et al., Nucleic Acids Res, 2015

### From stress coexpression clusters to stress coregulation gene network



Coexpression clusters for each category of stress

### Integration

Occurrence of pairs of coexpressed genes conserved in several stresses among the 18 considered stress categories



Coexpression network



Coregulation network

 Compared with random networks, only edges providing a FDR<1% were kept</li>

Filters

2) Only genes involved in triangles were considered as co-regulated

### Arabidopsis stress co-regulation network



4476 genes and 56487 co-regulation links

86% of the co-regulation links are supported by both biotic and abiotic stresses

- Scale-free network
  - Density = 0.006
  - Transitivity= 0.54
  - $\rightarrow$  Biological network
  - $\rightarrow$  Presence of gene clusters

### Stochastic Block Model (SBM) [WW87, NS01]

•  $Z_i$  independent hidden variables :

$$\blacktriangleright Z_i \sim \mathcal{M}(1, \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_K))$$

•  $Z_{ik} = 1$  : vertex *i* belongs to class *k* 

► X|Z edges drawn independently :

$$X_{ij}|\{Z_{ik}Z_{jl}=1\}\sim \mathcal{B}(\pi_{kl})$$

A mixture model for graphs :

$$X_{ij} \sim \sum_{k=1}^{K} \sum_{l=1}^{K} \alpha_k \alpha_l \mathcal{B}(\pi_{kl})$$



#### Maximum likelihood estimation

#### Log-likelihoods of the model :

• Observed-data :  $\log p(X|\alpha, \pi) = \log \{\sum_Z p(X, Z|\alpha, \pi)\}$  $\hookrightarrow K^N$  terms

 Expectation Maximization (EM) algorithm requires the knowledge of *p*(*Z*|*X*, *α*, *π*)

**Problem**  $p(Z|X, \alpha, \pi)$  is not tractable (no conditional independence)

Variational EM Daudin et al. [DPR08

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Variational EM Daudin et al. [DPR08] Graphical model and moral graph



### Model selection

#### Criteria

Since  $\log p(X|\alpha, \pi)$  is not tractable, we *cannot* rely on :

• 
$$AIC = \log p(X|\hat{\alpha}, \hat{\pi}) - M$$

► 
$$BIC = \log p(X|\hat{\alpha}, \hat{\pi}) - \frac{M}{2} \log \frac{N(N-1)}{2}$$

#### ICL

Biernacki et al.  $[BCG00] \hookrightarrow Daudin et al. [DPR08]$ 

Variational Bayes EM  $\hookrightarrow ILvb$ 

Latouche et al. [LBA12]

Others McDaid et al. [MDMNH13]



52 communities of 21 to 351 genes



Stability of the communities?

2674 genes in 43 communities describe the common response to stresses



### **Cross-validation procedure**

- For each stress category, create a network from the 17 others
- Find communities using mixture of graphs
- Comparison of these 18 results with the network built from all the categories



### Most communities with GO enrichments



### Functional validation of the communities



### Functional validation of the communities





Pictures by S. Domenichini

Functional validation of the communities



unknown function



Functional validation of the communities

Phenotyping of T-DNA mutants of 8 genes with unknown function

### Germination speed on 150mM NaCl

>3 biological replicates with 1 seed lot

Functional validation of the communities

2.0 Heat stress \*\*\* \*\*\* \*\* 3 days-old etiolated \* Hypocotyle length (cm) \*\*\* \*\* seedlings incubated at \*\*\* \*\*\* 5 44°C for 1h30 before 3 days of normal conditions. 3 independent exp. 1.0 0.5 Col0 ပ control S С С С S C C S C S S ഗ stressec

Phenotyping of T-DNA mutants of 8 genes with unknown function

Functional validation of the communities





# The backbone of plant stress response



### Identification of stress specific communities within the network



#### Introduction



FIGURE – An (hypothetic) email network between a few individuals.

#### Introduction



FIGURE – A typical clustering result for the (directed) binary network.

### Introduction [BLZar]



FIGURE – The (directed) network with textual edges.

### Introduction [BLZar]



**FIGURE** – Expected clustering result for the (directed) network with textual edges.

#### Context and notations

We are interesting in clustering the nodes of a (directed) network of *M* vertices into *Q* groups :

• the network is represented by its  $M \times M$  adjacency matrix A:

 $A_{ij} = \begin{cases} 1 & \text{if there is an edge between i and j} \\ 0 & \text{otherwise} \end{cases}$ 

if A<sub>ij</sub> = 1, the textual edge is characterized by a set of D<sub>ij</sub> documents :

$$W_{ij} = (W_{ij}^1, ..., W_{ij}^d, ..., W_{ij}^{D_{ij}}),$$

• each document  $W_{ij}^d$  is made of  $N_{ij}^d$  words :

$$W_{ij}^{d} = (W_{ij}^{d1}, ..., W_{ij}^{dn}, ..., W_{ij}^{dN_{ij}^{d}}).$$

### Modeling of the edges

Let us assume that edges are generated according to a SBM model :

each node *i* is associated with an (unobserved) group among *Q* according to :

 $Y_i \sim \mathcal{M}(\rho),$ 

where  $\rho \in [0, 1]^Q$  is the vector of group proportions,

the presence of an edge A<sub>ij</sub> between i and j is drawn according to :

$$A_{ij}|Y_{iq}Y_{jr}=1\sim \mathcal{B}(\pi_{qr}),$$

where  $\pi_{qr} \in [0, 1]$  is the connection probability between clusters *q* and *r*.

### Modeling of the documents

The generative model for the documents is as follows :

each pair of clusters (q, r) is first associated to a vector of topic proportions θ<sub>qr</sub> = (θ<sub>qrk</sub>)<sub>k</sub> sampled from a Dirichlet distribution :

 $\theta_{qr} \sim \operatorname{Dir}(\alpha)$ ,

such that  $\sum_{k=1}^{K} \theta_{qrk} = 1, \forall (q, r).$ 

the *n*th word W<sup>dn</sup><sub>ij</sub> of documents *d* in W<sub>ij</sub> is then associated to a latent topic vector Z<sup>dn</sup><sub>ij</sub> according to :

$$Z_{ij}^{dn}|\left\{A_{ij}Y_{iq}Y_{jr}=1,\theta\right\}\sim\mathcal{M}\left(1,\theta_{qr}\right).$$

then, given Z<sup>dn</sup><sub>ij</sub>, the word W<sup>dn</sup><sub>ij</sub> is assumed to be drawn from a multinomial distribution :

$$W_{ij}^{dn}|Z_{ij}^{dnk} = 1 \sim \mathcal{M}(1, \beta_k = (\beta_{k1}, \ldots, \beta_{kV})),$$

where *V* is the vocabulary size.

#### STBM at a glance...



FIGURE – The stochastic topic block model.

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FIGURE – The stochastic topic block model.

#### Inference

The full joint distribution of the STBM model is given by :

 $p(A,W,Y,Z,\theta|\rho,\pi,\beta)=p(W,Z,\theta|A,Y,\beta)p(A,Y|\rho,\pi).$ 

A key property of the STBM model :

- let us assume that Y is observed (groups are known),
- it is then possible to reorganize the documents  $D = \sum_{i,j} D_{ij}$  documents *W* such that :

$$W = (\tilde{W}_{qr})_{qr} \text{ where } \tilde{W}_{qr} = \left\{ W_{ij}^d, \forall (d, i, j), Y_{iq}Y_{jr}A_{ij} = 1 \right\},$$

- since all words in  $\tilde{W}_{qr}$  are associated with the same pair (q, r) of clusters, they share the same mixture distribution,
- and, simply seeing  $\tilde{W}_{qr}$  as a document *d*, the sampling scheme then corresponds to the one of a LDA model with  $D = Q^2$  documents.

#### Inference

Given the above property of the model, we propose for inference to maximize the complete data log-likelihood :

$$\log p(A, W, Y|\rho, \pi, \beta) = \log \sum_{Z} \int_{\theta} p(A, W, Y, Z, \theta|\rho, \pi, \beta) d\theta,$$

with respect to  $(\rho, \pi, \beta)$  and  $Y = (Y_1, \dots, Y_M)$ .





# 7 topics



bacteria

Nitrogen







## Conclusions of a biologist



# Conclusions of a biologist



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