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On Network Inference and Validation Methods

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Our BioSys Lab

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Our unit:

Bioinformatics and Systems Biology (Biosys)

Université de Liège, Belgium

Team biased towards large networks, machine learning and
algae...

Collaborating with three PhD students:

- Ngoc Pham (From Vietnam)
Expression-Based Transcriptional Networks
- Eoin Marron (From Ireland)
Chlamydomonas reinhardtii data-mining
- Pau Bellot (From Spain, co-tutelle with UPC)
Meta-network inference



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Notation

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- $X = (X_1, X_2, \dots, X_n)$: the set of n variables
- $X_k \in X$: one variable of the set
- $X_K \subset X$: a subset of variables
- $X_{-k} = X \setminus X_k$: set of variables without X_k
- X_{-K} : the set X without the subset of variables X_K
- $X_{i,j} = \{X_i, X_j\}$: two variables of the set X
- $X_{-(i,j)}$: set of variables X without X_i and X_j

Mutual Information (MI)



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Definition ([Thomas and Cover])

Let X_i and X_j be two (discrete) random variables, the mutual information between X_i and X_j is

$$I(X_i; X_j) = \sum_{x_i \in \mathcal{X}_i} \sum_{x_j \in \mathcal{X}_j} p(x_i, x_j) \log \left(\frac{p(x_i, x_j)}{p(x_i)p(x_j)} \right)$$

- Mutual information is a divergence between the joint and the product distribution.
- $I(X_i; X_j)$ is maximal if X_i or X_j is perfectly predictable from the other.
- $I(X_i; X_j) = 0$ if X_i or X_j are independent (unpredictable).



Conditional Mutual Information (CMI)

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Definition (*[Thomas and Cover]*)

Let X_i , X_j and X_k be three random variables, the conditional mutual information between two random variables X_i and X_j knowing X_k is

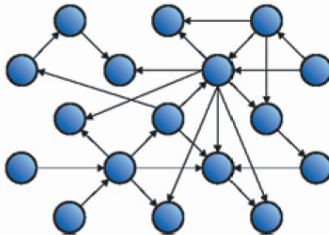
$$I(X_i; X_j | X_k) = I((X_i, X_k); X_j) - I(X_k; X_j)$$

- It measures the gain of information on X_j (or X_i) due to the other variable X_i (or X_j), when X_k is given.
- $I(X_i; X_j | X_k) \geq 0$ with equality iff X_i and X_j are conditionally independent given X_k .

Transcriptional Network

- $gene \rightarrow RNA \rightarrow protein$
- some protein (tf) can modify RNA production of target genes (tg)

⇒ Each cell has an encoded network (circuit) in DNA.



- Each node is a gene.
- An arc connects a regulator gene (tf) to a regulated one (tg).



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Problem Formalization



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- inputs X : $m \times n$ matrix, where x_{r_i} is the realization of gene X_i at measurement s_r
- output \hat{T} : list of triplets $(tf, weight, tg)$ of length $\#tf \times \#tg$

DATA	X_1	X_2	...	X_n
s 1	0.1	0.9	...	0.5
...
s m	0.2	0.3	...	0.8

 \Rightarrow

tf	w	tg
X_1	0.1	X_2
...
...
$X_{\#tf}$	0.9	$X_{\#tg}$



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Cause

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Definition (Cause [Neapolitan, 2003])

X_i is a *cause* of X_j , denoted by $X_i \rightarrow X_j$, if there exists a value $x_i \in \mathcal{X}_i$ such that setting $X_i = x_i$ leads to a change in the probability distribution of X_j .

In other words: causality creates a (bivariate) dependency between a cause and its effect.

$$X_i \leftrightarrow X_j \Rightarrow I(X_i; X_j) > 0$$

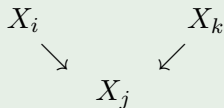
where $X_i \leftrightarrow X_j$ denote an *undirected causal link*, i.e., $X_i \rightarrow X_j$ or/and $X_i \leftarrow X_j$.

Assumption

$$X_j \leftrightarrow X_i \Rightarrow I(X_i; X_j) > 0$$

This bivariate dependency is true in most cases but not always: cancellation of two causal pathways, the XOR.

Example (XOR problem [*Neapolitan 2003*])



X_i	1	1	0	0
X_k	1	0	1	0
$X_j = X_i \oplus X_k$	0	1	1	0



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Indirect links

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- In most cases, $X_j \leftrightarrow X_i \Rightarrow I(X_i; X_j) > 0$
- Unfortunately, reverse is not true:
There are three cases of indirect interaction with three variables:
 - 1 $X_j \rightarrow X_k \rightarrow X_i$
 - 2 $X_j \leftarrow X_k \rightarrow X_i$
 - 3 $X_j \rightarrow X_k \leftarrow X_i$

Two of them typically lead to high $I(X_j; X_i)$

Direct Causality

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Definition (Direct cause [Neapolitan, 2003])

X_i is a direct cause of X_j if X_i is a cause of X_j and there is no other variable X_k such that once we know the value of X_k , a manipulation of X_i no longer changes the probability distribution of X_j .

It means:

two dependent variables are no longer dependent once given the direct cause.

$$\left. \begin{array}{l} X_i \rightarrow X_k \rightarrow X_j \\ X_i \leftarrow X_k \rightarrow X_j \end{array} \right\} \Rightarrow I(X_i; X_j | X_k) = 0$$

Direct causality (2)

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Equivalently: if there are no set of variables that cancel the dependency between two variables, then one of these variables is a direct cause of the other. More formally:

$$\forall X_K \subseteq X_{-(i,j)} : I(X_i; X_j | X_K) > 0 \Rightarrow X_i \leftrightarrow X_j$$

Implicit assumption of *causal sufficiency*, that is all the variables that cause at least two effects (two variables in the dataset) should also be present in the dataset:

$$\forall (X_i, X_j) \in X : \exists X_k, X_i \leftarrow X_k \rightarrow X_j \Rightarrow X_k \in X_{-(i,j)}$$

MRNET

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Network Inference Based on Variable selection
min-redundancy-max-relevance (mRMR) [Meyer et al., 2007]

$$X_i^{MRMR} = \arg \max_{X_i \in X-K} \left\{ I(X_i; X_j) - \frac{1}{|K|} \sum_{X_k \in X_K} I(X_i; X_k) \right\}$$

Bivariate approx. of $I(X_i; X_j | X_K) \rightarrow$ adapted to expression data

State-of-the-art

Method	RBN	ARACNe	Lasso	MRNET
Speed/Size	-	+	+	+
indirect arcs	+	-	+	+
non-linearity	+	+	-	+

Package: Bioconductor (5000+ downloads/year/since 2008)



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modENCODE project

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- Model Organism Encyclopedia Of DNA Elements (modENCODE) : the most comprehensive collections of functional datasets for a single organism: *D.melanogaster* [Celniker et al., Nature, 2009] (and *C.elegans*)
- 4 years of work from 50+ different institutions
- Kellis lab (CSAIL MIT + BROAD Institute) coordinating the integrative analysis to gain insights into the regulatory circuitry that controls gene expression in response to changing environments. [The modENCODE Consortium et al. Science 2010, genome Research 2012]

Problem

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Drosophila melanogaster data:

- Publicly available data:
 - list of >700 known tf
 - >14k genes
 - 12 Drosophila genomes
 - 139 known tf binding motifs
 - GO functional terms database
 - >1000 Protein-Protein Interactions
 - REDfly data
 - 2 "big" microarray datasets (Flyatlas + GSE6186)
- modENCODE data:
 - 2 RNAseq datasets
 - 2 histone modifications datasets
 - 76 tf-binding experiments (ChIP full genome)

→ **Transcriptional network?**



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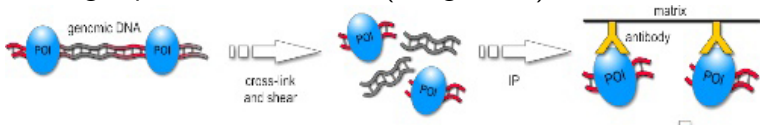
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ChIP-binding based network

Binding experiments for 76 tf's (full genome)



cond.	tf	chrom.	peakStart	peakEnd	intensity
t1	CG1674	chr2L	1	5954	0.9
...

→ threshold on intensity

but lots of non-functional binding (not intensity dependent)

Gene annotation file from flybase.org

name	chrom	txStart	txEnd	cdsStart	cdsEnd
CG1678	chr4	251355	266500	252579	266389
...

→ There is a link if binding near (+ - 500bp) of txStart



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ChIP-binding based network (2)

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For all tf - tg pairs, an edge weight is

- 0 if no binding evidence at 500 bp near $txStart$
- 0.1 if no data for a tf
- 1 if binding

→

tf	w	tg
X_1	0.1	X_2
X_i	0	X_k
...
$X_{\#tf}$	1	$X_{\#tg}$

Binding motif-based network



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From flybase.org

- DNA sequence
- 139 known tf binding motifs



→search (GREP) binding motif in the genome.

Problem: to many non-functional binding motifs

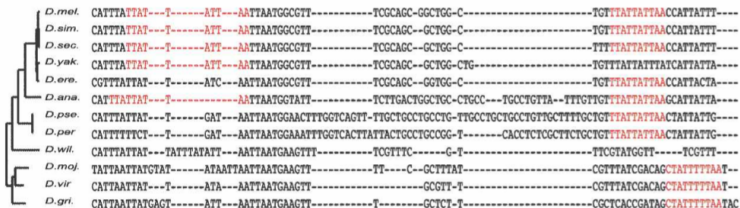
- gene annotation file

name	chrom	txStart	txEnd	cdsStart	cdsEnd
CG1674	chr4	251355	266500	252579	266389
...

→There is a link if tf motif near (+ - 500bp) of txStart

Binding motif-based network (2)

Use 12 *Drosophila* genomes with Branch Length Score (BLS) confidence [Kheradpour et al., gen.res., 2007]



BLS=25%

BLS=83%

→

tf	w	tg
X_1	0.1	X_2
X_i	0	X_k
...
$X_{\#tf}$	0.83	$X_{\#tg}$

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Expression based Networks



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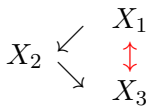
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Two steps:

- 1 Co-expression network: compute MI/correlation for all couples of genes
but false positive trends because of indirect links
Assume X_1 influence X_3 through X_2



Then $I(X_1; X_2)$ and $I(X_2; X_3)$ will be high
but also $I(X_1; X_3)$, hence it adds a false link between X_1
and X_3 .

- 2 Use an indirect-arc elimination algorithm on the correlation/MIM matrix.
 - ARACNE [Margolin et al, BMC Bioinfo, 2006]
 - MRNET [Meyer et al., BMC Bioinfo., 2008]



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Principle

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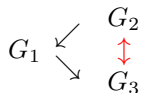
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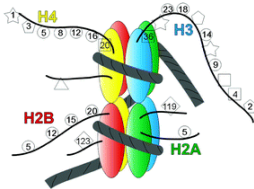
- Networks from sequence and/or tf binding
 - pro: physical connections (directed)
 - issue: elimination of non functional bindings
- Networks from expression and/or chromatin data
 - pro: functional connections (but undirected)
 - issue: elimination of indirect interactions



→ combine physical and functional networks to extract **direct functional interactions**

Chromatin regulation with histone modification

Chromatin can compact the genome up to 40000 times



- 5 families: H1, H2A, H2B, H3, H4
- The single-letter amino acid abbreviation (e.g., K for Lysine) and the amino acid position in the protein
- The type of modification: 4 modifications: me1, me2, me3, ac

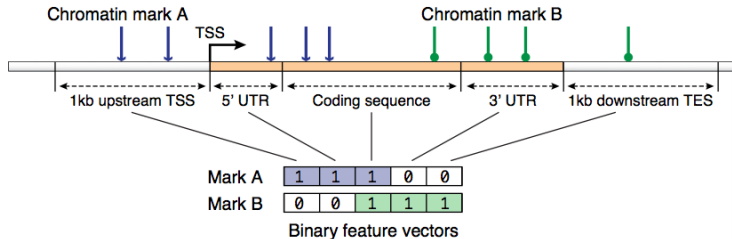
→ H3K4me1 denotes the monomethylation of the 4th residue (a lysine) from the start of the H3 protein.

51 distinct chromatin states suggests distinct biological roles (Ernst et al. Nature 2010).

Co-chromatin network

We have two datasets of measurements (ChIP)

- Ts: H3K4me1, H3K4me3, H3K9me3, H3K27me3, H3K27ac, H3K9ac
- Ct: H3K4me2, H4K16ac, H3K36me1, H3K36me3, H3K79me1, H3K79me2, H3K23ac, H3K18ac, H4K12ac, H4K5ac, H2BK5ac, H4K8ac.



Functional networks



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gene	M	A	R	K	1	M	A	R	K	2	...
tf	1	1	0	0	0	0	1	1	1	0	...
tg	1	0	0	0	0	0	1	1	1	1	...

squared Spearman correlation between

- tf and tg chromatin profiles (2 datasets)
→ 2 co-chromatin networks
- tf and tg expression profiles (3 datasets)
→ 3 co-expression networks
- 1 expression dataset kept for validation

→ 5 functional networks inferred + 2 physical networks
inferred (ChIP and motif)

Consensus Networks



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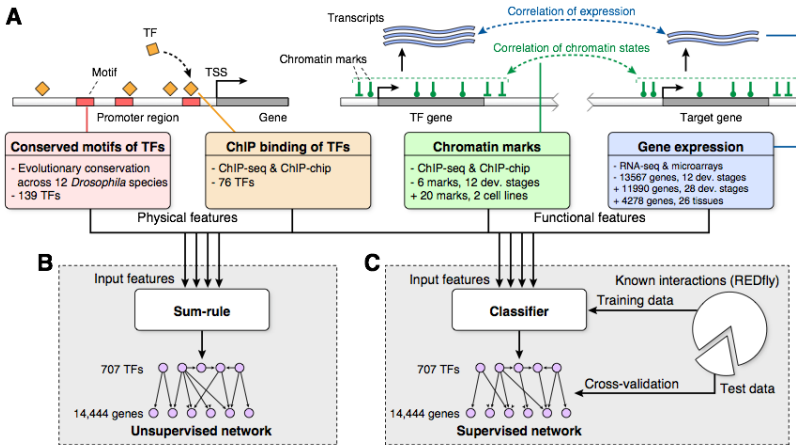
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Supervised Network



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Method: supervised logistic regression

- Weight w_{ij} from tf i to tg j , $w_{ij}^{output} = \frac{1}{1+e^{-m}}$

$$m = \alpha_0 + \alpha_{motif} w_{ij}^{motif} + \alpha_{ChIP} w_{ij}^{ChIP} +$$

$$\alpha_{chromtc} w_{ij}^{chromtc} + \alpha_{chromcl} w_{ij}^{chromcl} +$$

$$\alpha_{RNAseqtc} w_{ij}^{RNAseqtc} + \alpha_{arraytc} w_{ij}^{arraytc} + \alpha_{flyatlas} w_{ij}^{flyatlas}$$
- 10 fold cross-validation
- positive set: random sampling (with replacement) of 2k interactions of the 233 REDfly interactions
- negative set: random sampling of 2k interactions out of the 7k non-REDfly interactions
- fitting using iterative reweighted least squares
- final network: 318k edges (0.6 confidence)



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REDfly PR-Curves



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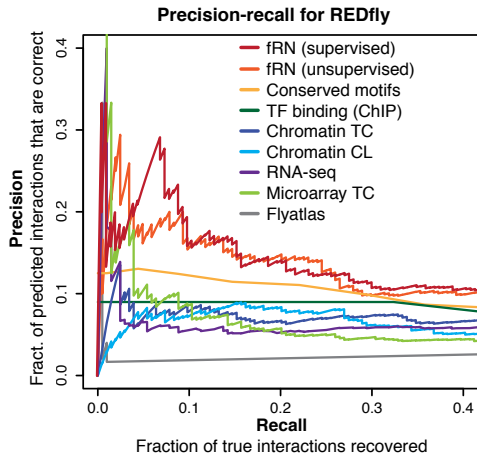
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Logistic regression weights: $\alpha_{motif,chromtc} = 2$,
 $\alpha_{ChIP,chromcl,RNAseq} = 1$, $\alpha_{array,flyatlas} = 0.4$

Structural properties: degree distributions



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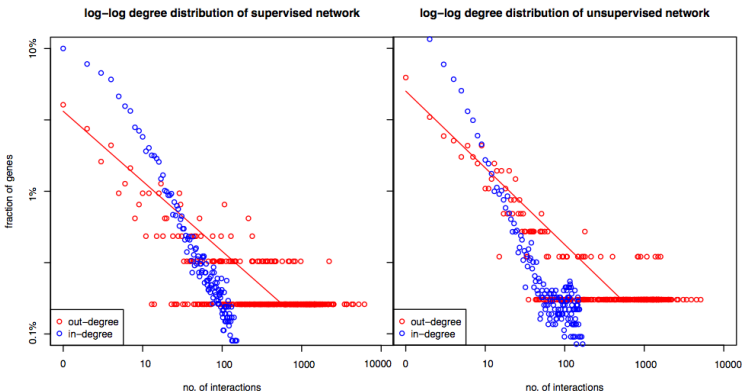
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Similar to *E.coli* and *S.Cerevisiae* known network topology

Most frequent three-nodes patterns



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





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Description	Network Motif Illustration	Statistical Significance	
		Fold Enr.	Z-score
Cross-regulating TFs co-targeting another TF (Double FFL)		17.919	104.23
		23.5	238.43
Cross-regulatory clique of TFs (Six FFLs)		2.891	10.65
		14.669	13.93
Cross-regulating TFs co-targeted by another TF (Double FFL)		1.989	23.72
		1.725	38.3
Cross-regulating TFs co-targeting a target gene (Double FFL)		1.594	69.01
		2.368	125.43
Feedback loop between three TFs		1.537	3.24
		1.154	2.62
Cross-regulating TFs creating a feed-forward and a feedback loop		1.349	7.52
		1.439	16.55

 Unsupervised
network

 Supervised
network


miRNA

Transcription
factor

Target gene

Biological Insights on co-targeted genes



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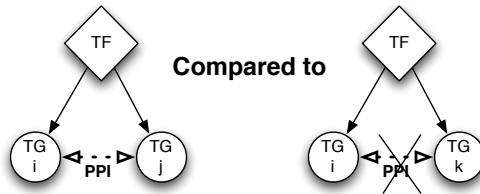
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Is the inferred network enriched in:



- 1 protein-protein interactions(PPI)
- 2 co-expressed in developmental cycle (RNAseq)
- 3 similar function profiles (GO terms)



Results

Fold enrichment of co-targeted genes

network	PPI	GO	RNAseq
motif	1.39	1.06	1.08
ChIP	1.24	1.23	1.46
unsupervised	1.53	1.44	3.07
supervised	1.58	1.55	3.62

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Results

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Our integrative networks outperform feature-specific networks

- PR-Curves on REDfly
- Enrichment of co-targeted genes on PPI, expression and GO terms

Our integrative networks fit known topological properties observed in *E.coli* and *S.cerevisiae*

- In-degree and out-degree
- Most frequent three-nodes patterns



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<http://homepage.meyerp.com>

Thank you!

Questions ?