Inférence de Réseaux bayésiens dynamiques à structure variant "progressivement" dans le temps

Etude de différents modes de partage d'information entre les segments successifs

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- Modelling regulatory networks from gene expression time series with DBN
- ARTIVA: Auto Regressive TIme VArying network
- Gradually time varying structure: segment information coupling
- Simulation study
- 8 Real data analysis

Recovering genes functions?





- up/down regulation
- retroaction, feedforwards loops...



\Rightarrow Complex dynamic system

• Objective: identifying this organisation in large scale.

Temporal gene expression data

• Microarrays:

→ simultaneous expression of several thousands of genes.



• Notations: we consider the stochastic process,

$$X = \{X_t^i; \forall i \in \{1, ..., p\}, \forall t \in \{1, ..., n\}\}$$

where X_t^i is the expression of gene *i* at time *t*,

What information extracting from expression profiles?

• Identifying coexpressed genes

 \rightsquigarrow coregulated genes? $\quad \rightsquigarrow$ same biological process?



• 2 main objectives:

- Which genes work together?
- At what time of the process?

DBN modelling of biological motifs

• A biological motif



• Dynamic Bayesian Networks (DBNs) ~ allow to model biological cycles



(Friedman et al. 1998, Murphy and Mian 1999)

DBN modelling

Assumptions

- (A_1) 1st order Markov process
- (A_2) 'simultaneous independence' given the past,

$$\forall t > 1, \forall i, j \in \mathbb{N}, \quad X_t^i \perp X_t^j \mid X_{t-1}.$$

• (A_3) time homogeneity



DAG $\tilde{\mathcal{G}}$ for a first order AR process

• AR(1) process: $\forall t \geq 1$, $X_t = AX_{t-1} + B + \varepsilon_t$, $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$



Proposition

If
$$\Sigma = Var(\varepsilon_t)$$
 is diagonal then $\tilde{\mathcal{G}}_A := \{X_{t-1}^j o X_t^i\} \Leftrightarrow a_{ij} \neq 0.$

DBN for a 1^{st} order auto-regressive process: AR(1).

• AR(1) process:
$$\forall t \geq 1, \ X_t = AX_{t-1} + B + \varepsilon_t, \ \ \varepsilon_t \sim \mathcal{N}(0, \Sigma)$$



Example:

$$\begin{array}{c} G^{1} \\ G^{2} \\ G^{2} \\ G^{3} \\$$

- MCMC, The Bayes Net Toolbox for MATLAB, Murphy (2001).
- Lasso, High-dimensional graphs and variable selection with the Lasso, Meinshausen and Bühlman (2006).
- Shrinkage, Learning causal networks from systems biology time course data: an effective model selection procedure for the vector autoregressive process, OpgenRhein and Strimmer (2007).
- **G1DBN**, Inferring dynamic genetic networks with low order independencies, Lèbre (2009).
- **SIMoNe**: *Statistical Inference for MOdular NEtworks*, Chiquet et al. (2009).

$\rightsquigarrow \mathsf{Time-homogeneous} \ \mathsf{DBNs}$

 \rightsquigarrow Remove (A_3) time homogeneity ?

Time-varying dynamic Bayesian network model

• Introducing changepoints where the network topology changes.



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• Undirected networks: Yoshida et al. (2005), Talih and Hengarten (2005), Xuan and Murphy (2007), Ahmed and Xing (2009).

• Directed networks:

• First attempts ;

 \rightsquigarrow Fujita et al. (2007): wavelet, fixed network structure

- \rightsquigarrow Rao et al. (2007): CP/edges estimated separately
- Most recent

→ Ahmed and Xing (2010): non-bayesian (parameter tuning, BIC, ...)

- → Robinson and Hartemink (2009, 2010): discrete network
- → Grzegorczyk and Husmeier (2009, 2011): fixed network structure

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 Auto Regressive TIme VArying (ARTIVA) network model Joint work with Gaëlle Lelandais, Frédéric Devaux, Jennifer Becq and Michael Stumpf
 BMC Systems Biology (2010)

- Our aims:
 - \rightsquigarrow Recover changepoints and edges simultaneously
 - \rightsquigarrow Gene specific segment transition
 - → Continuous data

ARTIVA network model definition.

p genes - n time points - k changepoint positions

For each gene i, $(1 \le i \le p)$,

• a changepoint vector

 $\boldsymbol{\xi}_{i} = (\xi_{i}^{1}, ..., \xi_{i}^{h-1}, \xi_{i}^{h}, ..., \xi_{i}^{K^{i}+1}) \subseteq \{2, ..., n+1\}$

- in each segment h, (for all $\xi_i^h \leq t < \xi_i^{h+1}$),
 - a set of s_i^h parents $G_i^h = \{j_1, ..., j_{s_i^h}\} \subseteq \{1, ..., p\}$
 - and a set of parameters $((w_{ij}^h)_{j \in \{0,...,p\}}, \sigma_i^h)$,

define the regression model,

$$X_i(t) = w_{i0}^h + \sum_{j \in G_i^h} w_{ij}^h X_j(t-1) + \varepsilon_i(t), \quad \varepsilon_i(t) \sim \mathcal{N}(0, \sigma_i^h).$$

- \rightarrow number of segments K_i ?
- \rightarrow number of edges s_i^h in each segment?

unknown dimension...

Priors

- Multiple changepoint model:
 - Number of changepoints $K_i \sim \mathcal{P}(\lambda)$
 - Changepoints vector $\xi_i | K_i \sim Uniform$
- Network model: (Andrieu and Doucet 1999)
 - Number of parents
 - Set of parents
 - Nois

• Noise variance
$$(\sigma_i^n)^2 \sim \text{Inverse Gamma}$$

• Regression coefficients $w_i^h | G_i^h, \sigma_i^h \sim \mathcal{N}\left(0, (\sigma_i^h)^2 \Sigma_{\mathcal{G}_i^h}\right)$
where $\Sigma_{\mathcal{G}_i^h} = \delta^{-2} D_{\mathcal{G}_i^h}^{\dagger}(x) D_{\mathcal{G}_i^h}(x)$ and $D_{\mathcal{G}_i^h}(x)$ has size $(\xi_i^h - \xi_i^{h-1}) \times (s_i^h + 1)$

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Hyperparameters:

- $\lambda, \Lambda \sim \mathcal{G}a(0.5, 1) \rightsquigarrow$ expected number of CP/edges
- $\delta^2 \sim \mathcal{IG}(2, 0.2) \rightsquigarrow$ expected signal-to-noise ratio

• From the AR model definition

$$P(x_i^h|G_i^h, a_i^h, \sigma_i^h) = \left(\sqrt{2\pi}\sigma_i^h\right)^{-\operatorname{length}(x_i^h)} \exp\left(-\frac{(x_i^h - D_{G_i^h}a_{G_i^h})^{\dagger} (x_i^h - D_{G_i^h}a_{G_i^h})}{2(\sigma_i^h)^2}\right)$$

Attractive feature (Andrieu and Doucet 1999): integration over parameters w and σ
 → analytical expression of P(k, ξ, G, λ, Λ, δ²|x)

Time-varying DBN inference with reversible jump MCMC

• Outline of the ARTIVA procedure:



→ Reversible jump MCMC (Green, 1995).
 → Model selection adapted from Andrieu and Doucet (1999).

Time-varying DBN inference with reversible jump MCMC

• 4 moves : Birth (b_k) , Death (d_k) , Position shift (u_k) , Regression model update (v_k) .

$$b_k + d_k + u_k + v_k = 1$$

Moves probability

$$b_k = c \min\left\{1, \frac{P_{\overline{k}}(k+1)}{P_{\overline{k}}(k)}\right\}, \ d_k = c \min\left\{1, \frac{P_{\overline{k}}(k-1)}{P_{\overline{k}}(k)}\right\}, \ u_k = \frac{1}{2}(b_k + d_k).$$

 \rightsquigarrow keep *c* small (less CP moves)

- Birth/death move
 - Acceptance ratio

 $R(\xi^+|\xi) = (Likelihood ratio) \times (Prior ratio) \times (Proposal ratio) \times (Jacobian)$

- (Likelihood ratio) ⊥ w, σ (Andrieu and Doucet, 1999)
 → Acceptance ratio based on the network structure only
- Acceptance probability $A(\xi^+|\xi) = \min\{1, R(\xi^+|\xi)\}$

~> Ensures reversibility

• Shift move: standard Metropolis-Hastings step

 $R(\tilde{\xi}|\xi) = (\text{Likelihood ratio}) \times (\text{Proposal ratio})$

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(Prior ratio =1)

Edge birth/death move

• Moves probabilities : birth $(b_{s_i^h})$, death $(d_{s_i^h})$, Parameter update (u)with $b_{s_i^h} + d_{s_i^h} + u = 1$

$$b_{s_i^h} = c_{s_i^h} \min\left\{1, \frac{P_{\overline{s}}(s_i^h+1)}{P_{\overline{s}}(s_i^h)}\right\}, \ d_{s_i^h} = c_{s_i^h} \min\left\{1, \frac{P_{\overline{s}}(s_i^h-1)}{P_{\overline{s}}(s_i^h)}\right\}$$

Acceptance ratio

$$R(\tilde{G}_i^h|G_i^h) = \text{(Likelihood ratio)} = \frac{P(x_i^h|\tilde{G}_i^h, \delta^2)}{P(x_i^h|G_i^h, \delta^2)}$$

as:

- Prior ratio \times Proposal ratio=1
- Jacobian=1

ARTIVA network inference with reversible jump MCMC

- Reversible jump MCMC procedure
 Generation of an ergodic Markov chain.
 - → Reversible Markov chain: detailed balance satisfied.
 - ~> Equilibrium distribution converges to the desired post-distribution,

 $P(k,\xi,s,G,w,\sigma|x).$

- R package ARTIVA freely available (http://cran.r-project.org)
- Simulation study + real data analysis in : "Statistical inference of the time-varying structure of gene-regulation networks" Lèbre S, Becq J, Devaux F, Lelandais G, Stumpf M. BMC Systems Biology 4(130) 2010

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Improvement : information coupling between segments

- Assumptions: independence of the different segments
 - risk of over flexibility
 - not realistic in many cases
- \bullet Introducing information coupling between segments \Rightarrow Various approaches
 - Inter-segment information coupling: Hierachical Bayesian model
 - Inter-node information: Hard coupling or Soft coupling
 - Prior distribution ?
 - Exponential distribution $\mathcal{E}xp(\beta)$
 - $\rightsquigarrow 1$ parameter
 - Binomial distribution $\mathcal{B}(a, b)$
 - \rightsquigarrow 2 parameters: 1 for edges similarity, 1 for non-edge similarity

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Hierachical Bayesian model for inter-segment information coupling

- Strength of the segment coupling: hyperprior Θ
- Level-2, level-3 hyperparameters: Ψ , Ω
- 2 schemes:

Hard coupling





Hard information coupling based on an exponential prior

 Prior modification (Werlhi and Husmeier, 2008) For all h ≥ 2,

$$P(G_i^h|G_i^{h-1},\beta) = \frac{e^{-\beta |G_i^h - G_i^{h-1}|}}{Z(\beta, G_i^{h-1})}$$

where

- β defines the strength of the coupling between G_i^h and G_i^{h-1}
- |.| the Hamming distance
- $Z(\beta, G_i^{h-1}) = \sum_{G_i^h \in \mathbb{G}} e^{-\beta \left|G_i^h G_i^{h-1}\right|}$ is a normalizing constant also known as the partition function
- When ignoring fan-in restriction (\overline{s}): $Z(\beta, G_i^{h-1}) = Z(\beta) = (1 + e^{-\beta})^p$

$$\Rightarrow P(G_i^h|G_i^{h-1},\beta) = \frac{e^{-\beta \left|G_i^h - G_i^{h-1}\right|}}{(1+e^{-\beta})^p}$$

Easy to integrate to ARTIVA (Lebre et al. 2010)

• Changepoint birth/death acceptance ratio ($G_i = \{G_i^h\}_{1 \le h \le K_i}$)

$$\mathbb{R}(\tilde{\xi}|\xi) = (\text{Likelihood ratio}) \times \frac{P(\tilde{G}_i)}{P(G_i)} \times (\text{Proposal ratio})$$

- Changepoint shift acceptance ratio : unchanged
- Edge birth/death ratio :

$$R(\tilde{G}_{i}^{h}|G_{i}^{h}) = \frac{P(x_{i}^{h}|\tilde{G}_{i}^{h})}{P(x_{i}^{h}|G_{i}^{h})} \times \frac{P(G_{i}^{h+1}|\tilde{G}_{i}^{h},\beta)P(\tilde{G}_{i}^{h}|G_{i}^{h-1},\beta)}{P(G_{i}^{h+1}|G_{i}^{h},\beta)P(G_{i}^{h}|G_{i}^{h-1},\beta)} \times \frac{Q(G_{i}^{h}|\tilde{G}_{i}^{h})}{Q(\tilde{G}_{i}^{h}|G_{i}^{h})}$$

- Parameter update : unchanged
- Additional MCMC step : sampling hyperparameter β
 With symmetric proposal probability,

$$R(\tilde{\beta}|\beta) = \frac{P(\tilde{\beta})}{P(\beta)} \prod_{i=1}^{p} \prod_{h=2}^{K_i} \frac{P(G_i^h|G_i^{h-1}, \tilde{\beta})}{P(G_i^h|G_i^{h-1}, \beta)}$$

 $P(eta) \sim \mathcal{U}[0,20]$ in our study.

Soft information coupling: $\beta \rightarrow \beta_i$

• For each node *i*,

$$P(G_{i}^{h}|G_{i}^{h-1},\beta_{i}) = \frac{e^{-\beta_{i}|G_{i}^{h}-G_{i}^{h-1}|}}{(1+e^{-\beta_{i}})^{p}}$$

• Common gamma prior $\mathcal{G}a(\kappa,\rho)$

$$P(\beta_i) = P(\beta_i | \kappa, \rho) = \beta_i^{\kappa-1} \frac{e^{-\beta_i/\rho}}{\rho^{\kappa} \Gamma(\kappa)}$$

We set

• $\rho = 0.1$ (mean $\mu = \kappa \rho$, variance $\sigma^2 = \kappa \rho^2$) • vague exponential prior on κ : $P(\kappa | \lambda_{\kappa}) = \lambda_{\kappa} e^{-\kappa/\lambda_{\kappa}}$ with $\lambda_{\kappa} = 10$ (prior ignorance) \rightsquigarrow coupling strength between node defined by the coefficient of variation $\frac{\sigma}{\mu} = \frac{1}{\sqrt{\kappa}}$ (small coefficient \rightarrow strong coupling)

Easy to integrate to ARTIVA (Lebre et al. 2010)

• Edge birth/death ratio :

$$R(\tilde{G_{i}^{h}}|G_{i}^{h}) = \frac{P(x_{i}^{h}|\tilde{G_{i}^{h}})}{P(x_{i}^{h}|G_{i}^{h})} \times \frac{P(G_{i}^{h+1}|\tilde{G_{i}^{h}},\beta_{i})P(\tilde{G_{i}^{h}}|G_{i}^{h-1},\beta_{i})}{P(G_{i}^{h+1}|G_{i}^{h},\beta_{i})P(G_{i}^{h}|G_{i}^{h-1},\beta_{i})} \times \frac{Q(G_{i}^{h}|\tilde{G_{i}^{h}})}{Q(\tilde{G_{i}^{h}}|G_{i}^{h})}$$

• Sampling hyperparameter β_i

$$R(\tilde{\beta}_i|\beta_i) = \frac{P(\tilde{\beta}_i)}{P(\beta_i)} \prod_{h=2}^{K_i} \frac{P(G_i^h|G_i^{h-1}, \tilde{\beta}_i)}{P(G_i^h|G_i^{h-1}, \beta_i)}$$

 Additional MCMC step: sampling κ With symmetric proposal probability,

$$R(\tilde{\kappa}|\kappa,\rho) = \frac{e^{-\tilde{\kappa}/\lambda_{\kappa}}}{e^{-\kappa/\lambda_{\kappa}}} \frac{P(\beta_i|\tilde{\kappa},\rho)}{P(\beta_i|\kappa,\rho)}$$

Binomial prior

$$P(G_i^h|G_i^{h-1},a,b) = a^{N_1^1[h,i]}(1-a)^{N_1^0[h,i]}b^{N_0^0[h,i]}(1-b)^{N_0^1[h,i]}$$

- N₁¹[h, i] is the number of edges in G_i^{h-1} matched by an edge in G_i^h
 N₁⁰[h, i] is the number of edges in G_i^{h-1} not matched by an edge in G_i^h
- $N_0^1[h, i]$ is the number of edges in G_i^h not matched by an edge in G_i^{h-1}
- $N_0^0[h, i]$ is the number of coinciding non edges in G_i^{h-1} and G_i^h
- Hyperparameter *a*, *b* prior :

$$P(a,b|lpha,\overline{lpha},\gamma,\overline{\gamma}) \propto a^{lpha-1}(1-a)^{\overline{lpha}-1}b^{\gamma}(1-b)^{\overline{\gamma}-1}$$

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First evalution : topology inference performance

- \Rightarrow Changepoint fixed at their true value
- \Rightarrow No structure change (only edges weights)
 - 10 networks of 10 nodes
 - number of parents : Poisson with mean $\lambda_{parent} = 3$
 - 4 segments of length 15
 - weights $w_{ij} \sim \mathcal{N}(0,1)$
 - noise $\varepsilon_i(t) \sim \mathcal{N}(0,1)$

First evaluation: no structure change



→ performance deteriorates with larger values of the hyperparameter
 → poor MCMC mixing and convergence...

Conclusion: large coupling strength affects the mixing of the Markov chain

Alternative MCMC scheme : multi-segments moves

- Remark: CP moves unchanged
- Target-node specific
- 2 steps (for target node *i*)
 - Sample one possible parent (j) for node i
 - **②** For each segment *h* of the K_i segments, flip the edge status between parent node and target-node *i* with probability q ($q = \frac{1}{2}$)
- Acceptance ratio (with $G_i = \{G_i^h\}_{1 \le h \le K_i}$)

$$R(\tilde{G}_i|G_i) = R_{Likelihood}(\tilde{G}_i|G_i) R_{prior}(\tilde{G}_i|G_i) R_{Proposal}(\tilde{G}_i|G_i)$$

•
$$R_{Likelihood}(\tilde{G}_i|G_i) = rac{P(x_i^h|\tilde{G}_i,\delta^2)}{P(x_i^h|G_i,\delta^2)}$$

- $R_{prior}(\tilde{G}_i|G_i) = \frac{P(\tilde{G}_i)}{P(G_i)}$
- Probability of proposing \tilde{G}_i given G_i : $\mathcal{Q}(\tilde{G}_i|G_i) = \frac{1}{p2^{K_i}}$ $\Rightarrow R_{Proposal}(\tilde{G}_i|G_i) = 1$

First evaluation: no structure change



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Binomial prior (no structure change)

• Area Under Precision Recall Curve (AUPRC)



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Binomial prior: original MCMC scheme + segment coupling



Binomial prior: with multi-segs moves



Simulations with structure changes

• Number of changes per node:

Poisson with mean $\lambda_{changes} = 0.25, 0.5, 1$

• Binomial prior (hard coupling): AUPRC



0.51	0.54	0.56	0.6	0.63	0.66	0.4				
0.54	0.56	0.59	0.61	0.65	0.65	0.5				
).54	0.59	0.6	0.63	0.64	0.65	0.6				
	0.6	0.62	0.64	0.65	0.64	0.7	a D			
0.56	0.6	0.62	0.64	0.65	0.64	0.7				
0.59	0.62	0.63	0.63	0.64	0.62	0.8				
).63	0.62	0.63	0.62	0.62	0.61	0.9				
0.4	0.5	9.0	0.7	0.8	6.0					
		,	.					. =		

Binomial prior (hard coupling): with multi-segs move



- DQC

Exponential prior (hard coupling): with multi-segs move

• Area Under Precision Recall Curve



Comprehensive simulation analysis



S. Lèbre sophie.lebre@lsiit-cnrs.unistra.fr ARTIVA network with segment coupling inference

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- Gene expression across the whole life cycle of *D. melanogaster* (Arbeitman et al., 2002)
 - 4028 genes
 - 67 successive time points
 - 4 temporal segments: Embryo Larva Pupa Adult
- Comparison with previous work on 11 genes involved in muscle development
 - TESLA (Ahmed and Xing, 2009)
 - Robinson and Hartemink 2009, 2010.

Real data application



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ARTIVA network with segment coupling inference

ARTIVA segments coupling

- Advantages over existing methods:
 - no need to discretize the data (\neq Robinson and Hartemink 2009, 2010)
 - allows structure changes (\neq Grzegorczyk and Husmeier, 2009, 2011)
 - all hyperparameters inferred from the data via a consistent Bayesian inference scheme (≠ Ahmed and Xing 2009)
 - includes four regularization coupling (\neq ARTIVA, Lebre et al. 2010)
- Detailed investigation of the hyperparameter inference
 ⇒ improved MCMC scheme for better convergence
- Difference hard/soft coupling seems negligeable in the investigated scenario...

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- Investigate hard versus soft coupling
- Investigate other functional forms for information sharing

e.g. recently Wang et al (2011) : exponential prior + additional parameter for sparsity prior

Our approach : sparsity with truncated Poisson distribution \Rightarrow explore the effect this additional sparsity parameter for gene network reconstruction

Joint work with:

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~ Submitted paper: "Non-homogeneous dynamic Bayesian networks with Bayesian regularization for inferring gene regulatory networks with gradually time-varying structure"